College-Level Mathematics Sample Questions

Answers with Explanations and Ratings

1. Rating: Medium

Explanation: In order to subtract these numbers, it would be beneficial to factor first. You cannot combine these two numbers as they are because the rules of exponents cannot be combined across addition or subtraction. However, if you first factor out the common factor, the problem is much simpler.

Answer: $2^{\frac{5}{2}}-2^{\frac{3}{2}}=2^{\frac{3}{2}}(2^{\frac{2}{2}}-1)=2^{\frac{3}{2}}(2-1)=2^{\frac{3}{2}}$ C

2. Rating: Medium

Explanation: In order to solve for x, you must isolate x and get it out of the bottom of the fraction.

Answer: $\begin{array}{c}\dfrac{1}{x}+\dfrac{1}{a}=\dfrac{1}{b}\\[2mm]\dfrac{1}{x}=\dfrac{1}{b}-\dfrac{1}{a}\end{array}$ Getting a common denominator, $\begin{array}{c}\dfrac{1}{x}=\dfrac{a}{ab}-\dfrac{b}{ab}\\[2mm]\dfrac{1}{x}=\dfrac{a-b}{ab}\end{array}$ so $x=\dfrac{ab}{a-b}$ E

3. Rating: Medium/Hard

Explanation: In order to solve this problem, you will use the method of completing the square:
1) Get everything with x on one side of the equation, and everything without x on the other side of the equation: $3x^2-2x=-7$

2) Factor out the leading coefficient: $3(x^2-\frac{2}{3}x)=-7$

3) Complete the square by cutting b in half, squaring it, then adding it to the end of the quadratic. Don't forget to add the same amount on both sides of your equation:

b = $\dfrac{-2}{3}$, half of this, squared is $\left(\dfrac{-1}{3}\right)^2=\dfrac{1}{9}$ so

$$3(x^2-\frac{2}{3}x+\frac{1}{9})=-7+3(\frac{1}{9})$$

4) Write the result in its perfect square form: $3(x-\frac{1}{3})^2=\dfrac{-20}{3}$

Divide by 3 to get:

Answer: $(x-\frac{1}{3})^2=\dfrac{-20}{9}$ E

4. Rating: Easy

Explanation: Parallel lines have the same slope, but different y intercepts. The only one of the choices

that has the same slope as y = 2x is

Answer: 2x - y = 4   C

5. Rating: Easy

Explanation: To get an equation of a line, you need the slope, which you can get using the given points. The formula for slope is: $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{2-0}{1-0}=2$ so the slope of the line is 2. Since it goes through the origin, its y intercept is 0. Therefore, using the form y = mx + b, the equation of the line is

Answer: y = 2x   A

6. Rating: Medium

Explanation: In this problem, there are 2 unknown pieces, the number of one bedroom apartments and the number of two bedroom apartments. It tells you that there are a total of 12 apartments. Since the problem is asking for the number of two bedroom apartments, you can build the work around that piece:

The number of two bedroom apartments = x
The number of one bedroom apartments = 12 - x (the remainder of the apartments)

To get the total rent, you need to add the rent from the two bedroom apartments and the rent from the one bedroom apartments:

Rent from the two bedroom apartments = 450x
Rent from the one bedroom apartments = 360(12 - x)
Total rent: 450x + 360(12 - x) = 4950

Solve for x: 450x + 4320 - 360x = 4950
                90x + 4320 = 4950
                90x = 630
                x = 7

Answer: 7   E

7. Rating: Medium/Hard

Explanation: In this problem, you are given the areas, but for fencing, you need the perimeters. Therefore, you can use the areas to find the sides.

For the larger square, $\begin{array}{c}s^2=125\\ s=\sqrt{125}=5\sqrt{5}\end{array}$ So the perimeter of the larger square is $4(5\sqrt{5})=20\sqrt{5}$

For the smaller square, $\begin{array}{c}s^2=5\\ s=\sqrt{5}\end{array}$ So the perimeter of the smaller square is $4(\sqrt{5})=4\sqrt{5}$

Therefore, the total fencing needed is $20\sqrt{5}+4\sqrt{5}$

Answer: $24\sqrt{5}$ C

8. Rating: Medium

Explanation: To solve for x here, you need to know how logs work. Logs undo exponential equations, so you can solve for x by working backward and turning this problem into the exponential equation. The base of the log becomes the base on your exponent:

$$If\ \log_{10}(x)=3, then\ 10^3=x$$

Answer: 1000   B

9. Rating: Medium

Explanation: This is a composite function. You need to plug g(x) into f(x):

$$f(g(x)) =\ 2\left(\frac{x-1}{2}\right)+1 =(x-1)+1=x$$

Answer: x   A

10. Rating: Medium

Explanation: This is a basic trigonometry problem, and it will help to draw a right triangle. If the angle is acute, the triangle is in the first quadrant, and all your trigonometric values are positive.

sin uses $\dfrac{opposite}{hypoteneuse}$, cos uses $\dfrac{adjacent}{hypoteneuse}$, which means you need the missing side of the right triangle to build cos. Use the Pythagorean Theorem, $a^2+b^2=c^2$:

$$1^2+b^2=2^2$$
$$1+b^2=4$$
$$b^2=3$$
$$b=\sqrt{3}$$

so $\cos(x)=\dfrac{\sqrt{3}}{2}$

Answer: $\cos(x)=\dfrac{\sqrt{3}}{2}$ D

11. Rating: Easy

Explanation: For this problem, you complete the factoring by factoring out the GCF:

5y(2y - 3) + (2y - 3) = (2y - 3)(5y + 1)

Answer: (5y + 1)(2y - 3)   B

12. Rating: Easy

Explanation: If you factor the quadratic, this is a perfect square, which means it will never be negative:

$$x^2 - 6x + 9 = (x-3)^2 \geq 0$$

Answer: This is negative for no real numbers.   E

13. Rating: Medium

Explanation: For this problem you must use the Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 - 5x - 1 = 0$ a = 1, b = -5, c = -1

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2(1)} = \frac{5 \pm \sqrt{25 + 4}}{2}$$

Answer: $\dfrac{5 + \sqrt{29}}{2}$ E

14. Rating: Easy

Explanation: The graph of y = x² is a parabola (U shaped) and is always positive, with a vertex at the origin. It will intersect the circle twice, since the circle's center is in the middle of the parabola.

Answer: Two   C

15. Rating: Easy

Explanation: The formula for slope is: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{r - 0}{0 - s} = -\dfrac{r}{s}$ so the slope of the line is 2. Since it goes through the origin, its y intercept is 0. Therefore, using the form y = mx + b, the equation of the line is

Answer: $-\dfrac{r}{s}$ A

16. Rating: Easy/Medium

Explanation: If you are lining up four things, there is a simple counting procedure:

Choices for first spot: 4
Choices for second spot: 3   4(3)(2)(1) = 24
Choices for third spot: 2   aka 4! = 24
Choices for last spot: 1

Answer: 24   E

17. Rating: Medium

Explanation: If you want to get $f^{-1}(3)$, you need to set $f(x) = 3$:

$$\frac{3x-1}{2}=3 \quad \text{Solve for x:} \quad \begin{aligned} 3x-1&=6 \\ 3x&=7 \\ x&=\frac{7}{3} \end{aligned}$$

Answer: 7/3   E

18. Rating: Easy/Medium

Explanation: If you use the rule to build the sequence, you can find a_3:

$$a_0=1$$
$$a_1=2a_0+2=2(1)+2=4$$
$$a_2=2a_1+2=2(4)+2=10$$
$$a_3=2a_2+2=2(10)+2=22$$

Answer: 22   E

19. Rating: Medium

Explanation: If you are choosing three things, there is a simple counting procedure:

Choices for first spot: 5
Choices for second spot: 4
Choices for third spot: 3

However, this does not take into account that the order doesn't matter, so this overcounts. To allow for this, you take out the overage:

$$\frac{5\times4\times3}{3!}=\frac{60}{6}=10$$

Answer: 10   D

20. Rating: Medium

Explanation: If you are taking a fraction to a power, if the power gets bigger the result gets smaller. Therefore if $a < b$, $f(a) > f(b)$.
Answer: $f(a) > f(b)$ E